Journal of Engineering Physics and Thermophysics, Vol. 77, No. 5, 2004

HYDRODYNAMICS AND HEAT EXCHANGE IN DISPERSED FLOWS

EQUALIZATION OF THE CONCENTRATION OF A SCALAR IMPURITY IN A FLOW-TYPE CHAMBER

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UDC 532.5

Experimental and theoretical investigations of the time of equalization of the concentration of an impurity in a rectangular flow-type chamber have been carried out. It has been shown that the process of equalization of the concentration with time is exponential in character. The characteristic equalization time has been computed using the theory of turbulent diffusion. Theoretical results describe experimental regularities with an accuracy of about 10%. The value of the coefficient of turbulent diffusion for different configurations of flows in the chamber has been obtained from a comparison of experimental and calculated results.

The rate of equalization of the concentration (temperature) of a scalar impurity in the volume of a flow-type chamber is of interest for many technical applications. Among the problems in which this parameter is important are filling of the combustion chamber with fuel mixture, removal of toxic impurities from rooms by forced ventilation, propagation of the contaminant in flowing water reservoirs, etc. As the characteristic time of establishment of the stationary pattern of distribution of a scalar impurity in such systems one usually selects t_r determined as

$$t_r = V/Q \; .$$

However experiments show that the actual time in which a system reaches the stationary regime is several times longer than t_r in many cases. Nonetheless, it is noteworthy that the above evaluation of the establishment time is independent of the shape of the chamber and the character of flow. The present work seeks to develop a method of calculation of the time in which the concentration of the scalar impurity reaches the stationary regime that takes more accurate account of the pattern of flow and the geometry of the chamber.

It is clear that a stable channel with a higher-than-average concentration of the scalar impurity rapidly appears in the flow system between the points of entry and exit of the flow. In the remaining regions of the chamber, secondary vortex flows develop; by these flows the impurity from the main flow propagates throughout the volume of the chamber (Fig. 1). We use the theory of turbulent diffusion ([1–3] and references therein) to describe the propagation of the scalar impurity. We note that a complete hydrodynamic description of the velocity and concentration fields in problems on equalization seems unnecessary. In our approach, it is sufficient to know only the dependence of the coefficient of turbulent diffusion D on the basic parameters of flow.

Let us denote the height, length, and width of the chamber as H, L, and W respectively. We guide the ordinate axis along L. We introduce the hydraulic radius of the chamber R as

$$R = \frac{WH}{W + H}$$

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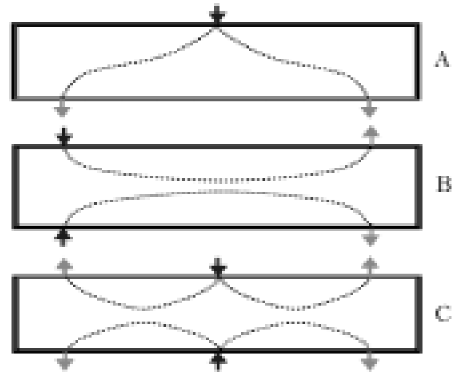


Fig. 1. Three investigated configurations of flows in the chamber.

and the Reynolds number Re on the hydraulic diameter of the chamber

$$\mathrm{Re} = 2\rho QR / (\mu HW) \ . \tag{1}$$

If the Re number is fairly large (about 1000), a natural turbulent flow is generated in the chamber. The characteristic velocity of secondary flows v_p in the chamber is determined by the value of the average flow rate

$$v_{\eth} \sim Q / (HW) \ .$$

For smaller Reynolds numbers the kinetic energy of secondary flows in the chamber volume is determined by the kinetic energy of the inflowing turbulent jets. In this case the characteristic velocity of the secondary flows is $v_p \sim Q/(d_{in})^2$, where d_{in} is the diameter of the inlet. It is precisely for this case that the experiments described below have been carried out.

To obtain the dependence of the coefficient of turbulent diffusion D on the velocity of secondary flows we represent the coefficient of turbulent diffusion as the integral of the correlation function of the velocity field [1]. After the approximate integration we have

$$D = C\Lambda v_{\eth} \ ,$$

where Λ is the integral scale of the velocity field [4] and C is the function that must be determined experimentally. Since the integral scale of the velocity field has the order of the smallest geometric dimension of the chamber, i.e., $H = \min\,(H,\ W,\ L)$, then

$$D = CHv_{\eth} = CHQ / (d_{in})^2 \ . \tag{2}$$

In the general case the function C in the expression for the coefficient of turbulent diffusion can be written as

$$C = F\,(W/H\,,\ L/H\,,\ \mathrm{Re}) \ , \tag{2$'$}$$

where the dimensionless function F depends on dimensionless arguments.

In the present work, we give results of experimental investigation of turbulent diffusion in a rectangular chamber with different combinations of the arrangement of inlets and outlets. In the two regimes, use is made of the collision of the jets inside the chamber that accelerates the process of equalization. The rate of equalization of the concentration of the scalar impurity is calculated based on the approximate solution of a two-dimensional nonstationary problem on turbulent diffusion. Tentative results of calculation of the turbulent diffusion of soot particles in a flow-type chamber (for configuration A in our notation) and experimental data on the dynamics of reaching the stationary regime have been published in [5].

Experimental Investigation. The experimental chamber measures 600 × 160 × 100 mm. Six unions for taking pipelines are symmetrically arranged on the lateral walls of the chamber. By changing the site of connection of

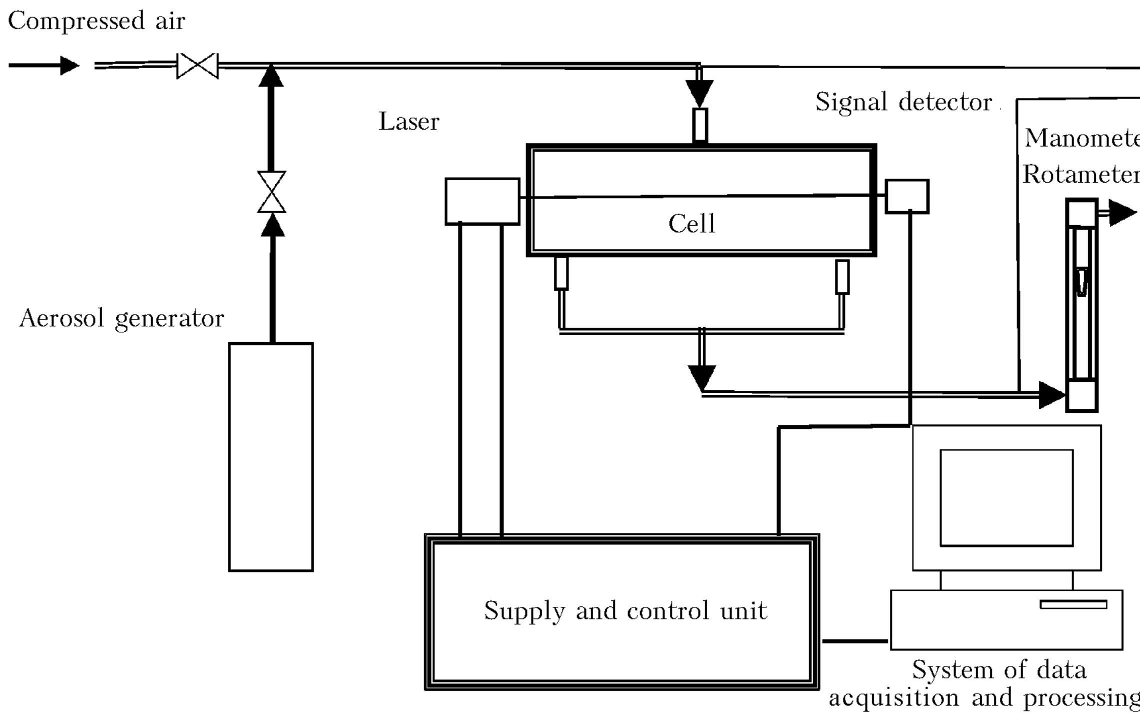


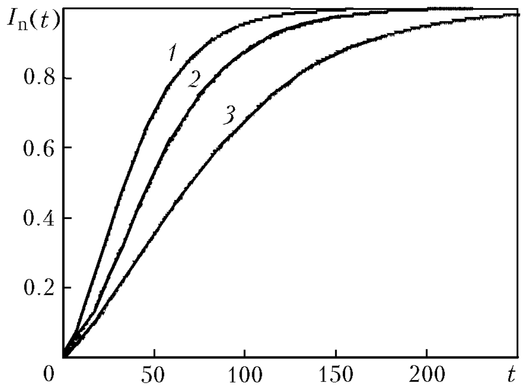
Fig. 2. Diagram of the experimental setup.

inlet and outlet pipelines we can obtain different configurations of the flow inside the chamber. In the present work, we have investigated three configurations of the flow, denoted subsequently as A, B, and C and diagrammatically shown in Fig. 1. The diameters of the flow sections of the unions were equal to 8 mm, whereas the distances between them were equal to 190 mm. One wall of the chamber was made of transparent Plexiglas, which enabled us to observe the development of a vortex structure in the process of filling of the chamber with air. To visualize the flow we added to the air an aerosol of the ester of diethylhexyl sebacic acid (DES), produced by an AGF 2 aerosol generator (PALAS GmbH, Karlsruhe). This generator ensured a constant flow rate of air and enabled us to smoothly control the concentration of the aerosol. The diameter of aerosol particles, as our experiments had shown, was less than one micrometer. Once the chamber had been filled with the aerosol, it was blown with pure air. In the process of blowing, we recorded the change in the intensity of the laser beam introduced into the cell through the optical windows along the longitudinal axis of symmetry. The optical system of diagnostics of the parameters of aerosol particles used for the experiments has been described in [6]. The operating principle of this system is based on measurement of the attenuation of the intensity of radiation from three lasers with dissimilar wavelengths in scattering of light on solid particles in a two-phase flow. The system consists of the laser supply and control unit, the laser head, the radiation detector, and the computer system of data acquisition and processing. A diagram of the experimental setup is presented in Fig. 2.

The experiments seek to investigate the dependence of the time of total removal of the aerosol from the chamber on the configuration of the flow, the flow rate of air, and the volume of the chamber. It has been found that in the process of blowing of the chamber, the normalized signal $I_n(t)$ of the detector of laser radiation increased exponentially with drop in the average concentration of the aerosol along the optical path:

$$I_n(t) = \frac{I(t) - I_0}{I_{max} - I_0} = 1 - \exp(-t/\tau) , \quad (3)$$

where $I(t)$, I_{max}, and I_0 are respectively the running, maximum, and minimum values of the signal and τ is the constant appearing in this expression. The time of establishment of the equilibrium concentration T was taken to be 3τ. Figure 3 shows the change in the intensity of the normalized signal of the detector for three different flow rates of air (configuration A).



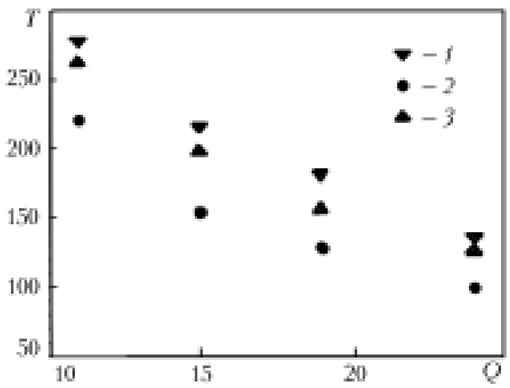


Fig. 3. Normalized intensity of the optical system I_n vs. time t: 1) Q = 24; 2) 19; 3) 11 liters/min. t, sec.

Fig. 4. Experimental dependence of the time of establishment of equilibrium concentration on volumetric flow rate Q: 1) configuration A; 2) B; 3) C. T, sec; Q, liter/min.

TABLE 1. Time of Establishment of the Stationary Density of the Scalar Impurity in the Chamber as a Function of the Chamber Volume and the Volumetric Flow Rate of Air

Q, liter/min	$V^{\mathrm{I}} = 9.6$ liter			$V^{\mathrm{II}} = 5.95$ liters			$T^{\mathrm{I}}/T^{\mathrm{II}}$
	$\langle\tau\rangle$, sec	T^{I}, sec	$(T/t_{\mathrm{r}})^{\mathrm{I}}$	$\langle\tau\rangle$, sec	T^{II}, sec	$(T/t_{\mathrm{r}})^{\mathrm{II}}$	
11	92	276	5.3	55	165	5.03	0.60
15	71	213	5.5	43	129	5.4	0.61
19	60	179	5.9	34	102	5.4	0.57
24	44	132	5.5	25	75	5.0	0.57

The times of establishment of the equilibrium concentration in the chamber as functions of the value of the flow rate of air that have been determined experimentally for different configurations of the flow are given in Fig. 4. For all the investigated configurations of the flows, the establishment time is in inverse proportion to the volumetric flow rate Q or the Reynolds number Re.

We also investigated the influence of the chamber's height H on the time of establishment of the equilibrium concentration. The volume of the chamber V^{I} was reduced with two 19-mm-thick plastic inserts arranged symmetrically. The height and reduced volume of the chamber V^{II} were 0.62 of their initial value ($V^{\mathrm{II}} = 0.62V^{\mathrm{I}}$). The results of the experiment on the influence of the height on the establishment time for configuration A are given in Table 1. An inverse proportion between the establishment time and the volumetric flow rate holds for the chamber of reduced volume. The ratio of the establishment times is virtually constant and somewhat lower than the ratio of the volumes of the chambers.

In the course of the experiments, we carried out visual observations of the process of equalization of the concentration of the aerosol in the chamber's volume. From the observations of the flow pattern it was found out that, once the aerosol has been fed to the chamber, a main channel of the flow is formed between inlets and outlets; this main channel sharply differs from the remaining volume of the chamber in concentration. The subsequent equalization of the concentration is relatively slow. The results of the visual observations of the equalization time quantitatively coincide with the results of measurements of the attenuation of laser radiation with a relative accuracy of approximately 10%.

The main regularities of propagation of the impurity (exponential growth, inverse proportion to the flow rate, and influence of the chamber's height) found in the experiment will be obtained within the framework of the following mathematical model of turbulent diffusion of a scalar impurity in the chamber.

Mathematical Model. Let us use the equation of turbulent diffusion with a distributed source whose role is played by the main channel. The intensity of the source depends on the product $D\nabla n(x, y, z, t)$, where the gradient must be calculated at the boundary of the main channel. Since the position of the boundary is determined inadequately, we replace $D\nabla n(x, y, z, t)$ by the approximate expression $\pi Dd(n_0 - n(x, y))f(x, y, z)/(RHW)$, where n_0 is the density of the scalar impurity in the main flow and d is the diameter of the main channel in which the main flow containing the scalar impurity streams. The dimensionless function $f(x, y, z)$ has a zero value everywhere except at the boundary of the region occupied by the main channel, where it is equal to unity. We assume that d is substantially smaller than R.

We introduce the coordinate system so that its origin coincides with the center of the chamber. Then the diffusion equation for the eigenfunctions $n(x, y, z, t)$ that describes the propagation of the scalar impurity with density from the main channel has the form

$$\partial_t n\,(x, y, z, t) = D\,[\partial_{xx} n + \partial_{yy} n] + Dd\,(n_0 - n)\,\pi f\,(x, y, z)/(RHW)\,. \tag{4}$$

If $n_0 > n$, we have the regime of filling of the chamber with the impurity; if $n_0 < n$, we have the regime of emptying of the chamber. Both these cases have been investigated experimentally. The stationary regime of operation of the flow-type chamber occurs when $n_0 = n$.

The boundary conditions to Eq. (4) reflect the condition of impermeability of the chamber walls Γ:

$$\nabla n \big|_{\Gamma} = 0\,. \tag{5}$$

We use the Galerkin method to determine the characteristic time of reaching the stationary state [7]. We select the product of the first eigenfunctions of Eq. (4) without a source that exactly satisfy boundary conditions (5) as the fundamental mode for description of the density of the impurity. As a result, we have the following expression for the fundamental mode $\varphi(x, y, t)$:

$$\varphi\,(x, y, z, t) = A\,(t, z)\cos\left(\frac{\pi x}{H}\right)\cos\left(\frac{\pi y}{W}\right). \tag{6}$$

Consequently, the density of the scalar impurity $n(x, y, z, t)$ can be written in the form of the sum $\varphi(x, y, t)$ and higher modes. It is well known that the contribution of higher modes can be disregarded for problems of determination of the characteristic time of the diffusion process [4].

To obtain analytical evaluations we assume that n_0 is constant with time and inside the main channel of the convective flow. By virtue of this assumption, the dependence on the geometric dimension of the chamber L does not occur in the solution. Substituting expression (6) into the linear differential equation (4) instead of the density of the impurity n and then using the Galerkin procedure [7], we obtain the equation for the amplitude $A(t)$:

$$\partial_t A = -\pi^2 D\left[\frac{1}{H^2} + \frac{1}{W^2} + \frac{k_1}{4\pi^2 k_0}\right]A + k_2/k_0\,, \tag{7}$$

where the coefficients k_0, k_1, and k_2 appearing in the ordinary differential equation (7) are determined as

$$\begin{aligned} k_0 &= \int\!\!\int \cos^2\left(\frac{\pi x}{H}\right)\cos^2\left(\frac{\pi y}{W}\right)dxdy = 0.25HW\,, \\ k_1 &= D\,\frac{\pi d}{RHW}\int\!\!\int f\,(x, y, z)\cos^2\left(\frac{\pi x}{H}\right)\cos^2\left(\frac{\pi y}{W}\right)dxdy\,, \\ k_2 &= Dn_0\,\frac{\pi d}{RHW}\int\!\!\int f\,(x, y, z)\cos\left(\frac{\pi x}{H}\right)\cos\left(\frac{\pi y}{W}\right)dxdy\,. \end{aligned} \tag{8}$$

The approximate computation of the integrals yields the following estimate:

$$k_2 \sim \frac{\pi D n_0 d^3}{RWH}, \quad k_1 \sim \frac{\pi D d^3}{RWH}.$$

By virtue of selection of the basis function in the form (6), the main contribution to the value of the corresponding integrals is yielded by the values of the sources in the central part of the chamber. In other words, those parts of the channel that are at a large distance from the longitudinal axis of symmetry make a small contribution to the values of k_2 and k_1. The stationary solution of Eq. (7) A_s has the form

$$A_s = \frac{4k_2}{\pi^2 HWD\left[\frac{1}{H^2} + \frac{1}{W^2} + \frac{k_1}{\pi^2 HW}\right]}. \tag{9}$$

We note that the quantity A_s is in direct proportion to n_0 and is independent of D.

We seek the solution of the ordinary differential equation (7) with the initial condition $A(0) = 0$ for the regime of filling of the chamber. The evolution of the amplitude of the fundamental mode will be written as

$$A(t) = A_s\,[1 - \exp(-t/\tau)].$$

We seek the solution of (7) with the initial condition $A(0) = A_s$ for the regime of cleaning of the chamber. The expression for the evolution of the amplitude of the fundamental mode will be as follows:

$$A(t) = A_s \exp(-t/\tau).$$

We emphasize that, when the times are fairly long, the entire density of the scalar impurity has the same dependence. Thus, the experimental results given above and partially published in [5] confirm the character of the chamber reaching the stationary operating regime. The characteristic time of filling of the chamber τ is equal to

$$\tau^{-1} = \pi^2 D\left[\frac{1}{H^2} + \frac{1}{W^2} + \frac{k_1}{\pi^2 HW}\right], \tag{10}$$

the dependence of τ on the parameter k_2 is absent in expression (10) and, as the numerical evaluations for our experiments show, the dependence of τ on the parameter k_1 is very weak. In numerical evaluation, the diameter of the main channel is assumed to be equal to the diameter of the inlet. From the mathematical viewpoint, τ^{-1} is the lowest eigenvalue of the diffusion problem (4) with boundary condition (5).

As follows from (10) and (2), the dependence of the characteristic time τ on the volumetric rate of flow of the gas through the chamber can be represented by the relations $\tau \sim D^{-1}$ or $\tau \sim Q^{-1}$. The dependence of τ on the geometric dimensions is not so obvious and has the form

$$\tau \sim H\,(d_{in})^2/[Q\,(1 + H^2/W^2)]$$

for our experimental conditions.

Discussion of Results. We have investigated experimentally and theoretically the process of equalization of the spatial distribution of the scalar impurity in a rectangular chamber for three configurations: A, B, and C (see Fig. 1). It has been shown that for the regime of cleaning of the flow-type chamber, the density of the scalar impurity changes exponentially with time; the characteristic equalization time found experimentally exceeds the natural scale (which is the residence time) several times. Regime B in which flow with a maximum Reynolds number occurs corresponds to the minimum characteristic equalization time. By virtue of the symmetry of flow, the Reynolds number is two times smaller for configurations A and C than that for configuration B. From the data presented in Fig. 4 it is clear that the equalization times coincide, in practice (with an accuracy of about 6%) for configurations A and C. Processing of experimental data has shown that the quantity C varies in the range $(25–35)\cdot 10^{-6}$ for our experiments. We note that the collision of jets in configurations B and C, apparently, increases the amplitude of velocity pulsations.

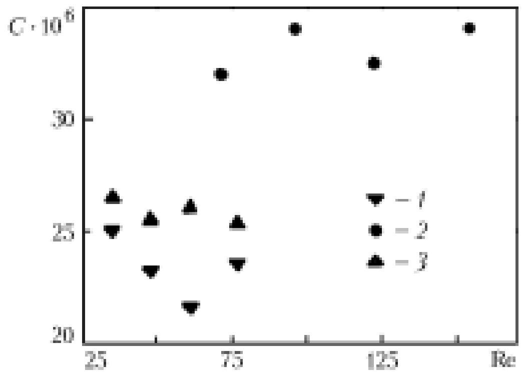


Fig. 5. Dependence of the constant C on the Reynolds number: 1) configuration A; 2) B; 3) C.

The effective coefficient of turbulent diffusion for the entire flow-type chamber depends on the Re number, as the analysis of the experimental data on the basis of the developed theory has shown. Indeed, secondary flows in our experiments are initially generated by jet flows, but the rate of their attenuation substantially depends on the Re number: the higher the number, the slower is the attenuation of the secondary flows [4].

A theoretical analysis in the approximation of turbulent diffusion has demonstrated that the process of equalization of the concentration of the scalar impurity exponentially depends on time, whereas the characteristic equalization time has a dependence on the geometric parameters of the chamber typical of diffusion problems. For the correctness of comparing the results obtained and the experimental results we must compute the time of establishment of secondary vortex flows in the chamber volume from the experimental values of the establishment time. We can take the time of establishment of the main flow to be equal to t_r with a sufficient degree of accuracy.

The experiments with a changed height of the chamber have confirmed that the effective coefficient of turbulent diffusion D depends on the Reynolds number Re. For the experiments described above (see Table 1) the Reynolds number increases by approximately 10% with decrease in the chamber's height and a constant gas flow rate. Therefore, for the chamber with a smaller height the establishment time expressed by t_r decreases. In particular, for a flow rate of 19 liters/min the ratio of the experimental values (presented in Table 1) is equal to $\tau(H)/\tau(0.62H) = 1.8$. The calculated value of this ratio is equal to 1.44, if we disregard the dependence of the coefficient of turbulent diffusion on the Re number; the relative accuracy is equal to 20%. With allowance for C as a function of the Re number (Fig. 5), the calculated value of the ratio will have an accuracy of about 10%.

Processing of experimental data with (10) and (2) enables us to obtain the following expression for the coefficient of turbulent diffusion in the chamber having the shape of a rectangular parallelepiped:

$$D = 2C \frac{\mu}{\pi\rho} \frac{H^2 W}{d_{in}^2 R} \mathrm{Re}\,.$$

For numbers $\mathrm{Re} \geq 150$ we have $C \approx 35 \cdot 10^{-6}$. We note that the μ/ρ ratio actually determines the coefficient of molecular self-diffusion. A complex dependence of the coefficient of turbulent diffusion on the geometric dimensions of the chamber is obvious from (10). Generalization of the expression for the characteristic turbulent-diffusion time (9) to chambers of another geometry presents no fundamental problems.

Thus, the time of establishment of a uniform distribution of the scalar impurity in the chamber T can be expressed by the residence time and the characteristic diffusion time as $T = 3\tau + t_r$.

The authors express their thanks to the DAAD Agency (Germany) for partial support of the work.

NOTATION

$A(t, z)$, amplitude of the fundamental mode, m^{-3}; C, numerical coefficient; D, coefficient of turbulent diffusion, m^2/sec; d, diameter of the main channel, m; d_{in}, inlet diameter, m; H, chamber height, m; I_n, normalized dimensionless value of the optical signal; L, chamber length, m; n, density of the scalar impurity, m^{-3}; Q, volumetric flow rate, m^3/sec; R, hydraulic radius, m; Re, Reynolds number determined in terms of R; T, time of establishment of the equilibrium concentration, sec; t, time; t_r, residence time, sec; V, chamber volume, m^3; v_p, velocity of secondary flows, m/sec; W, chamber width, m; x, y, z, space coordinates; Λ, integral scale of the velocity field, m; μ and ρ, viscosity and density of the medium, kg/(m·sec) and kg/m^3; τ, characteristic equalization time. Subscripts: in, inlet; max, maximum value; n, normalized; p, pulsation; r, residence; s, stationary; 0, initial value.

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